A METHOD FOR DETERMINING THE THERMAL DIFFUSIVITY

OF SOLID PROPELLANT ROCKET FUELS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

SUMMARY

The purpose of this investigation was to examine a proposed method for determining the thermal diffusivity of solid propellant rocket fuels. This method employs a rapid experimental procedure providing data from which thermal diffusivity may be obtained by a simplified calculation procedure.

An unsteady-state heat transfer system was constructed which permitted the elevation of the temperature at one end of a solid, cylindrical sample of rocket propellant initially at a uniform temperature throughout. The sample was surrounded by nested pipe insulation and enclosed in a box to minimize radial heat losses and to provide essentially unidirectional heat flow in the sample. A cylindrical heater assembly was inserted through an access port in the sample container until it made contact with one end of the sample. The temperature-response histories at various locations in the sample were measured by means of thermocouples embedded in the sample.

The temperature history data obtained during an experimental run were used to calculate thermal diffusivity by a modification of a simple error function calculation method for temperature response to a step-function rise in surface temperature. The results obtained by this simplified calculation procedure were compared with results obtained by a much longer calculation procedure employing an exact-solution equation derived for the experimental conditions. This exact equation was solved by trial and error on an IBM 650 Digital Computer.

The results calculated by the simplified calculation procedure deviated more than 10 per cent from the mean value when averaged over a long period of run time. The results of the exact-solution procedure verified the adverse effects of heat losses from the sample approximately five minutes after the start of the temperature rise at the response temperature location in the sample. The results obtained by the simplified calculation procedure within this five minute data-run period were within 10 per cent of the results obtained by the exact-solution procedure. The diffusivity values calculated by the exact solution provided reproducibility within 5 per cent. No evaluation of the method could be made using low-conductivity samples of known thermal diffusivity due to the unavailability of these materials in the required geometric configuration.

The recommendation is made that additional investigation be conducted with an improved system in which heat-loss effects are reduced and that an evaluation of the method be determined by using samples whose thermal diffusivities are known or can be determined by measuring the thermal conductivities, specific heats and densities of the samples separately.

CHAPTER I

INTRODUCTION

Thermal diffusivity is defined as the ratio of the thermal conductivity of a substance to the product of its density and heat capacity and it is expressed in units of area per unit of time. This combination of properties is an important proportionality constant in the equation for unsteady-state heat conduction in materials and, as a result, it has an important role in many practical heat transfer calculations. There is currently vital interest in the heat transfer characteristics of solid propellant rocket fuels among those people who design vehicles powered by these fuels and those who manufacture solid propellants. As a result of this interest, there is a demand for thermal diffusivity data on the many types of solid propellants that are being developed.

New, experimental, solid propellants are being designed continuously with many different compositions obtained by varying the types and the amounts of the constituents of which they are composed. With so many of these "prototype" fuels in continuous development, it is important that rapid methods for determining thermal diffusivity of the fuels be available. The purpose of this investigation is to examine a method for determining the thermal diffusivity of solid propellant rocket fuels which employs a rapid experimental procedure providing data from which thermal diffusivity may be obtained by a simplified calculation procedure. The original investigation was sponsored by Rohm and Haas Company, Redstone Arsenal

Research Division, Huntsville, Alabama.

Thermal diffusivity may be obtained by several different methods. A rather common one involves measuring separately the three properties, thermal conductivity, density and heat capacity, which, taken appropriately, comprise the thermal diffusivity of the sample. The experimental procedures associated with determining each of these properties are complicated by many difficulties arising from the unique problems involved in handling rocket fuels. In addition, the experimental procedures are laborious and time-consuming; thus, a method for the direct determination of thermal diffusivity is desirable.

An experimental method for determining thermal diffusivity directly would, of necessity, require the measuring of temperature-time relationships at various locations in the sample during an unsteady-state heat transfer process; there is no thermal diffusivity term in the steady-state heat transfer equation. The boundary conditions existing during the unsteady-state procedure determine the type of mathematical methods that must be employed to solve the unsteady-state conduction heat transfer equation. Several examples of these mathematical solutions are provided by Jakob (1) and Schneider (2). Chung and Jackson (3) successfully determined thermal diffusivity of plastic materials and double-base solid propellant samples. In their method, the apparatus and mathematical solution were based upon radial heat transfer in a solid, cylindrical sample of sufficient length to be considered, theoretically, infinitely long. The use of the "infinite" sample satisfied one boundary condition demanded by the mathematical solution employed.

Another method, having a simpler but less accurate mathematical solution, is based upon axial heat flow in a solid sample of sufficient length and insulated on all but one surface such that it can be considered a semi-infinite solid. Using this method, one surface of the sample having an initially uniform temperature throughout is suddenly brought to and maintained at some higher temperature for the duration of the test, i.e., the surface temperature is a step function of time. The temperature response to this sudden elevation of surface temperature at some location in the sample a given distance from the surface and after some duration of time from the surface temperature rise is given by the simple error function response to a step function boundary condition. If this response temperature is measured, it provides a value for thermal diffusivity, as can be seen from the step function solution. This solution, as given by Jakob (1), is,

$$\frac{t_x - t_i}{t_s - t_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha\tau}}\right)$$

Since values for the complementary error function are tabulated in several sources such as Schneider (2) and Carslaw and Jaeger (4), the function term $\frac{x}{2\sqrt{\alpha\tau}}$ and, hence, thermal diffusivity can be determined.

Extremely difficult, if not impossible, is the practical achievement of a step-function rise in the surface temperature of a sample. In addition, measuring the temperature at the surface of contact between the sample and the heat source is equally difficult.

Positioning a thermocouple in the sample as near as possible to the heated surface is easier. If the temperature at this point is considered to be the surface temperature, the location of the response-temperature thermocouple must be measured from the thermocouple near the surface. Since the surface temperature at this point somewhat removed from the actual heated surface would probably not exhibit a step-function rise, a surface temperature value obtained by graphically integrating the curve of surface temperature versus time from zero time to the "equation time", τ , and dividing this value by τ would probably provide sufficiently accurate results.

This investigation examines the latter method experimentally and attempts an evaluation of the accuracy of the resulting thermal diffusivity values by comparing them with results obtained using a more exact, but considerably more complex, mathematical solution based upon the Duhamel's equation. This solution provides a more accurate temperature response expression for the specific surface temperature experimentally encountered. A development of this equation is provided in Appendix B.

CHAPTER II

DESCRIPTION OF APPARATUS

The apparatus used in this investigation was an unsteady-state heat transfer system. It was designed to permit the sudden elevation of the temperature at one end of a cylindrical sample of propellant, initially at a uniform temperature throughout, and the determination of the temperature history at various locations in the sample after the elevation of the surface temperature. A schematic diagram of the apparatus is shown in Figure 1. Photographs of the system and various components are presented in Figures 2 through 5. These figures show that the apparatus consisted of three categories of components: the sample and its container, the heating system, and the temperature measuring system.

The Sample and Sample Container.--The propellant samples were solid cylinders one inch in diameter and thirty inches long. These were manufactured by the project sponsor. Thirty-gauge, chromel-alumel thermocouples were embedded at specified locations during the casting of the samples. One thermocouple was located as near one end of the sample as possible and the temperature history measured at this point was used as the surface-temperature function. Another thermocouple was located approximately one inch from the end thermocouple. A third thermocouple was located approximately five inches from the end thermocouple. All three thermocouples were positioned as accurately as possible at the same radial

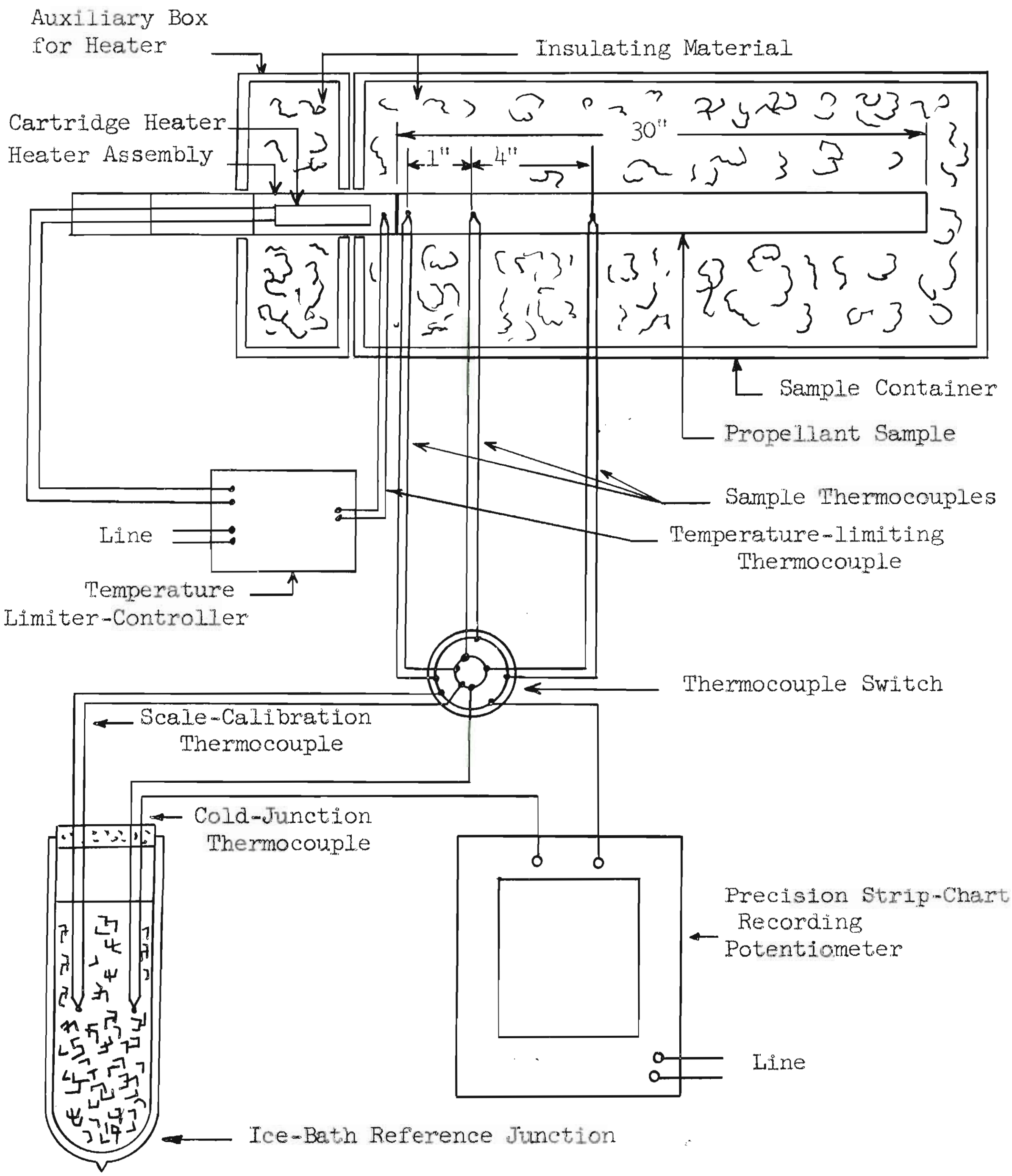


Figure 1. Schematic Diagram of Experimental Apparatus

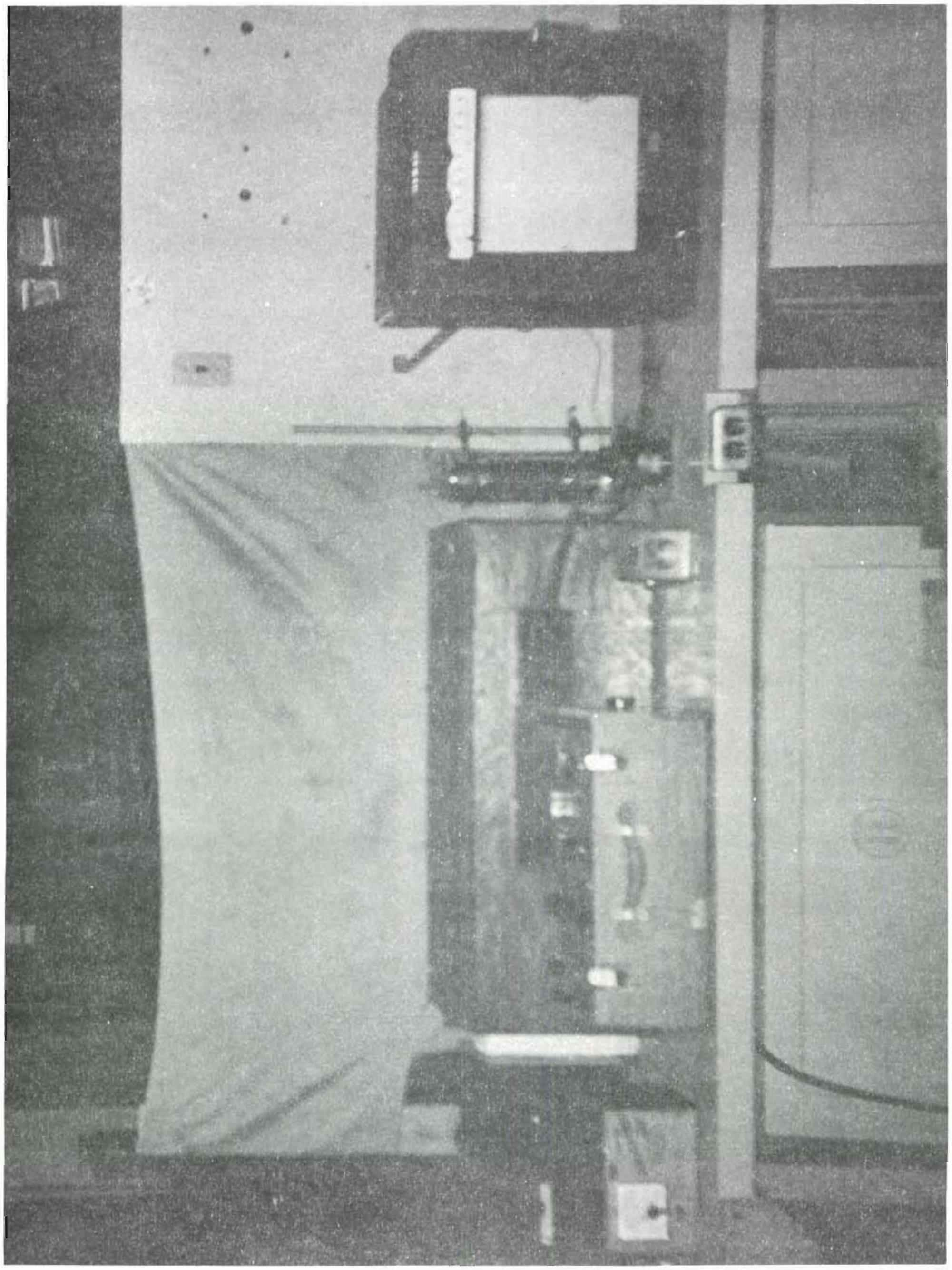


Figure 2. Overall View of Experimental Apparatus.

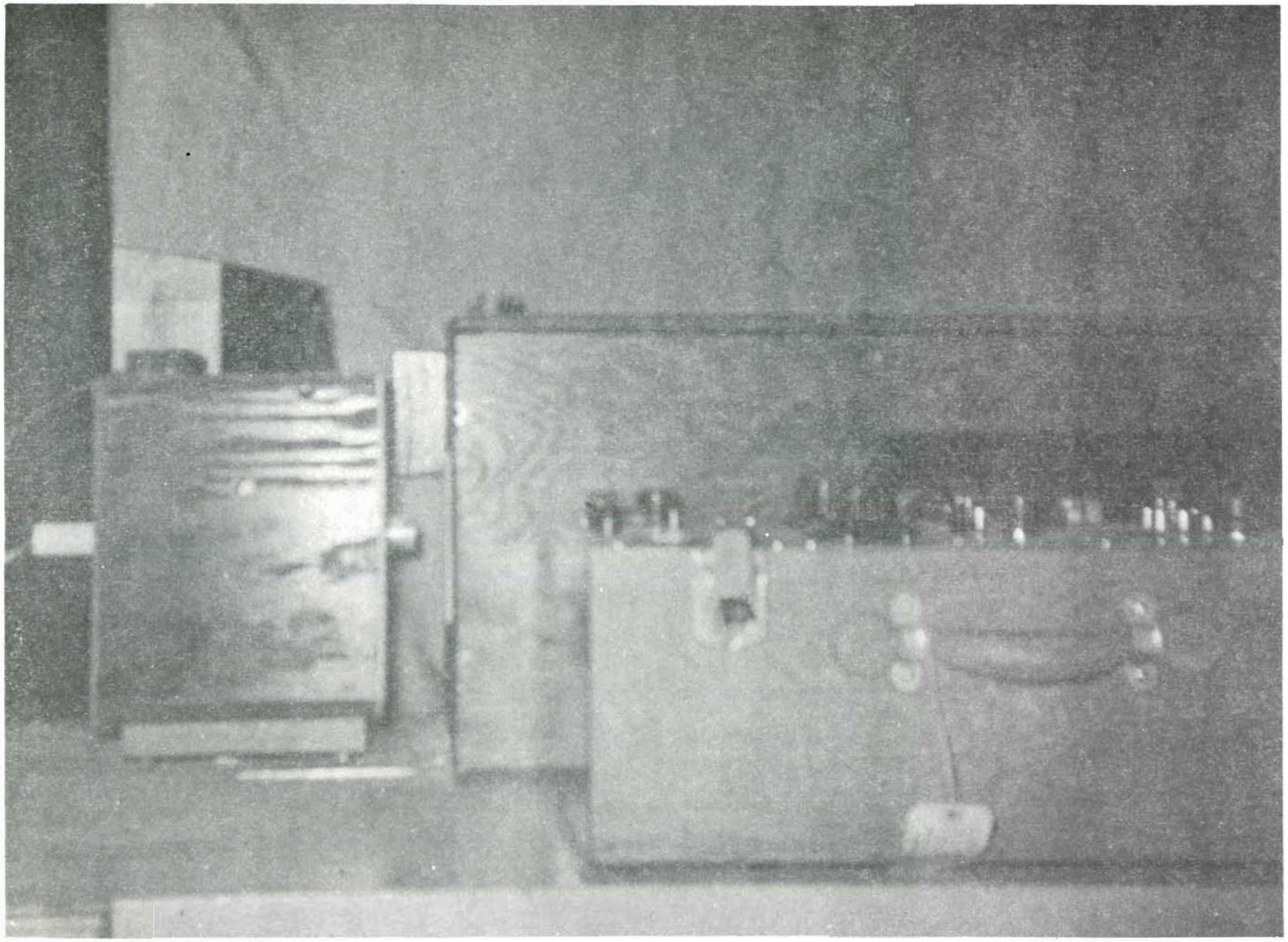


Figure 3. Heater Assembly and Auxiliary Box Positioned at the Access Port of the Sample Container.

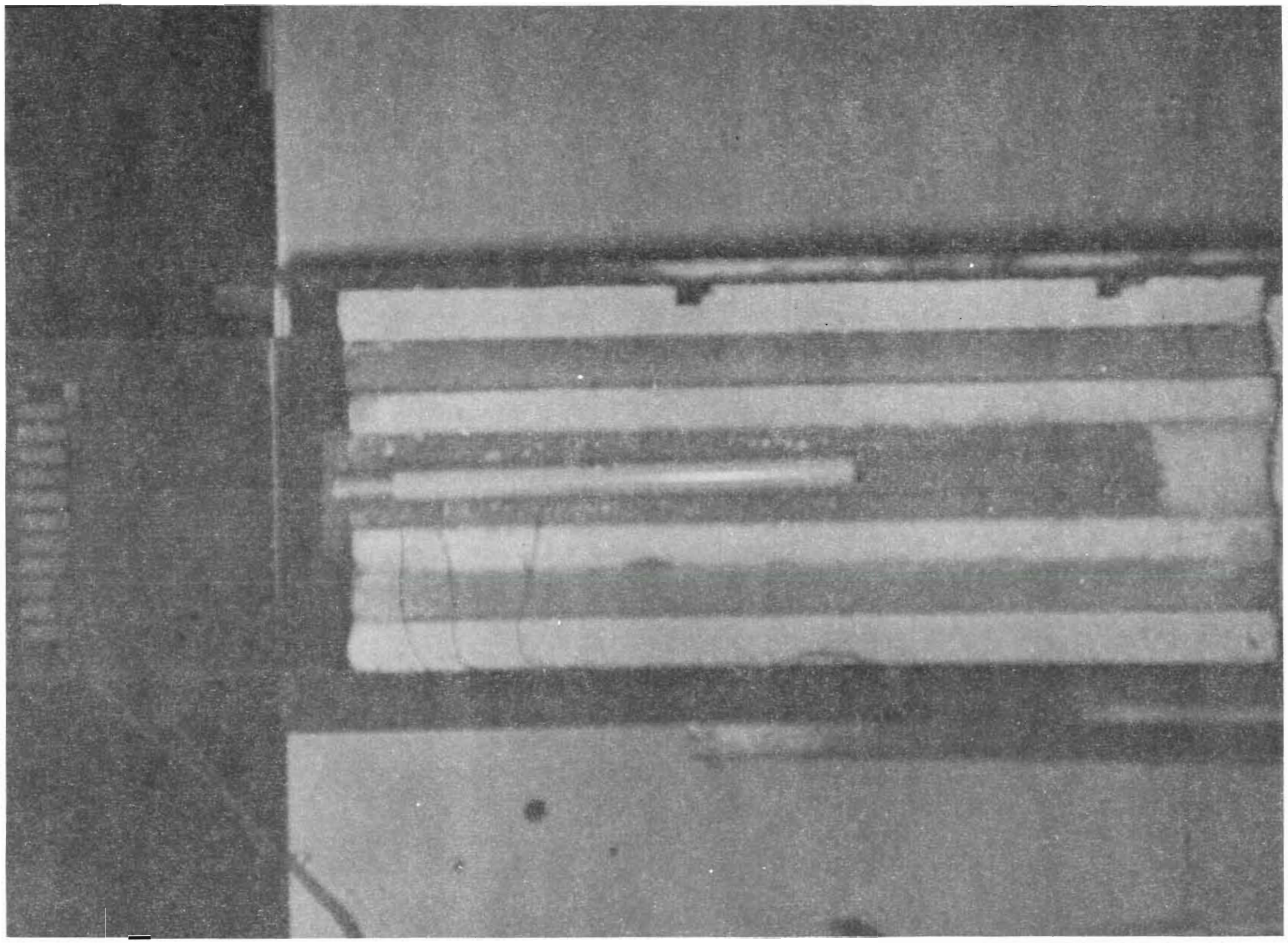


Figure 4. Heater Assembly in Contact with One End of the Sample in the Sample Container.

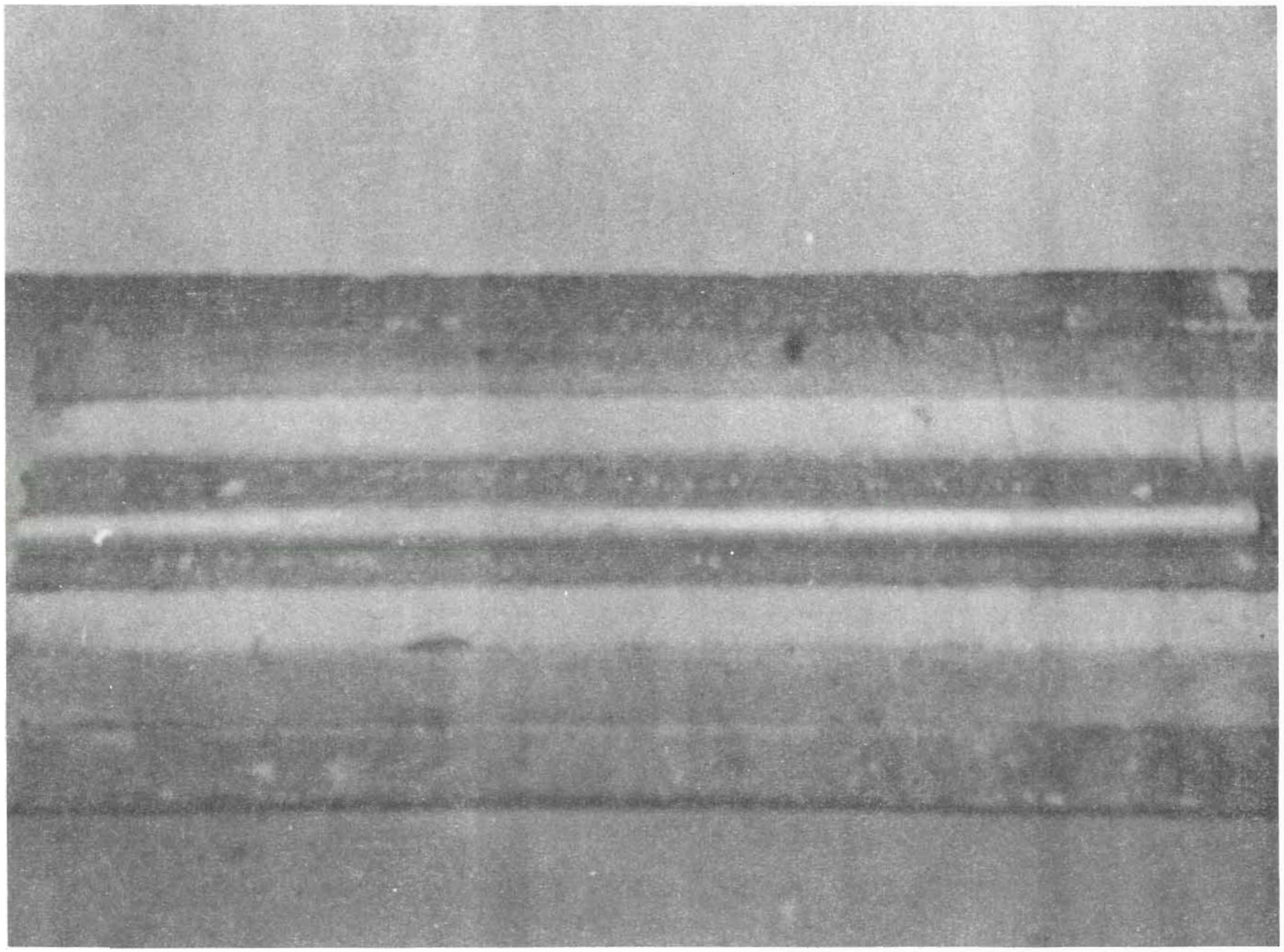


Figure 5. Detail View of Sample and Sample Thermocouple Positions in Nested Pipe Insulation.

depth, i.e., the radial axis of the sample.

The sample was surrounded by nested, Kaylo pipe insulation to provide a total radial insulation thickness of six inches. The thermocouple wires were lead through the seams of the pipe insulation halves. A plywood box just large enough to enclose the sample-in-insulation assembly was constructed and a removable lid was attached to permit rapid change of samples. The sample thermocouple leads were inserted through a slot in the side of the box matched with the seams of the insulation. These leads were then connected to a standard thermocouple terminal block mounted on the side of the box just below the slot. The slot and terminal block were provided with a draft-preventing cover with a single hole just large enough to accommodate the leads from the temperature measuring system which were connected with the sample thermocouple leads at the terminal block.

One end of the plywood sample container box was made of transite insulating material. A hole one-inch in diameter was bored in the center of the transite side such that the hole lined up exactly with the hollow core in the nested insulation in which the propellant sample was placed. The propellant sample was positioned in the insulation and container such that the end of the sample in which the three thermocouples were embedded faced the hole in the transite end of the container. This hole served as the heater access port to the sample surface to be heated during an experimental run.

The interior of the sample container was lined with asbestos paper and several vent holes were provided in the container lid as fire and explosion safeguards in the event of accidental ignition of the propellant samples.

The Heating System.--The heating system consisted of a heater assembly, an auxiliary box for the heater and a temperature limiter-controller. The heater assembly was a rod of laminated design as shown in Figure 6. The heating section, or laminate, was an aluminum rod one inch in diameter and three inches long. A hole was bored in this aluminum cylinder from one end to a depth of 2-5/8 inches to just fit a 250-watt, electric cartridge heater. The other end of the aluminum rod was polished to form a good heater contact surface. A tiny radial hole was drilled near the polished end surface to accommodate a 30-gauge, iron-constantan, glass-insulated thermocouple which was used to monitor the temperature of the heater assembly at the contact surface. A groove was milled from the thermocouple hole down the length of the rod to the end opposite the heating surface. The iron-constantan thermocouple lead was pressed down into this groove and cemented into place with porcelain heater cement which filled the groove. When the cement hardened, this area was sanded to restore the uniform, cylindrical shape to the heater rod. This was done to facilitate insertion of the rod into the sample container.

The end opposite the heater surface was counter-bored to mate with the shoulder on the second laminate of the assembly. This section was a cylinder, also one inch in diameter, of Colorlith insulation material. The thermocouple lead and heater wires were accommodated by a hole bored through the entire length of the section. This laminate served as an insulator between the aluminum heater section and the final laminate, a

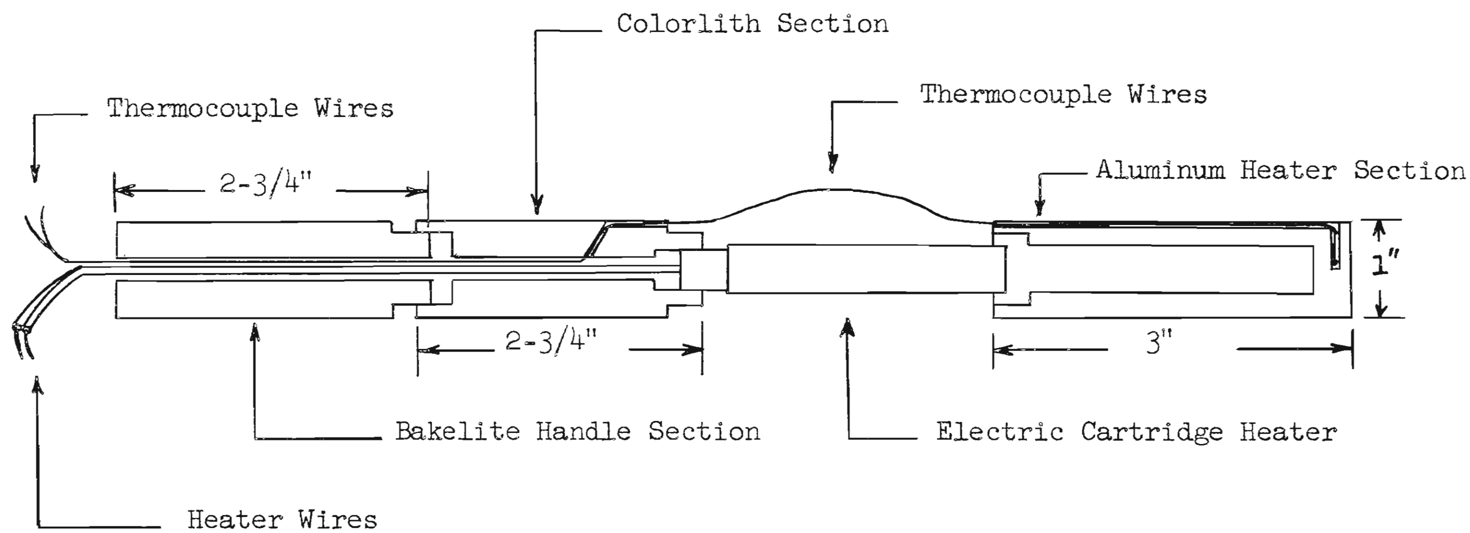


Figure 6. Exploded View of Heater Assembly.

one-inch diameter rod of bakelite. This section was also center-bored to accommodate the electrical and thermocouple leads. The bakelite section served as a handle with which the assembly was easily inserted into the sample container during a data run.

The thermocouple and electrical leads were connected to a Sim-ply-trol pyrometer, temperature limiter-controller which was used to prevent the surface of the heater from exceeding a safe temperature with respect to the propellant sample.

The heater assembly was contained in an auxiliary box and insulated by nested Kaylo pipe insulation. The auxiliary box was constructed of plywood except for the end of the box through which the heated surface of the heater assembly was thrust during insertion of the heater into the sample container. This end, as in the construction of the sample container, was made of transite with a hole bored to match the heater assembly with the access port in the sample container. The auxiliary box was mounted on ball-type casters to facilitate rapid positioning of the heater assembly to the access port of the sample container.

The Temperature Measuring System.--The temperature histories (i.e., temperature-time functions) at the three thermocouple locations in the sample were measured by a Leeds and Northrup Speedomax, Type G, Series 60000, precision recording potentiometer. Leads from the terminal block on the sample container connected the sample thermocouples through a Leeds and Northrup, 10-point, thermocouple selector switch to the recording potentiometer. Since the recording potentiometer was a single-point recorder, the thermocouple selector switch permitted selection and reading of any of the three sample thermocouples during a data run.

A cold-junction reference thermocouple for the potentiometer was inserted in a Dewar ice bath. The potentiometer was calibrated for a zero reading before each data run by means of another thermocouple also located in the ice bath.

CHAPTER III

EXPERIMENTAL PROCEDURE

In preparation for a data run, a sample propellant was enclosed in the pipe insulation to reduce radial heat losses and provide an essentially unidirectional, axial heat flow in the sample to obey, as closely as possible, the calculation boundary condition of heat flow in a semi-infinite slab. The sample thermocouple leads were connected to the terminal block on the side of the sample container.

The precision recorder was placed in operation and calibrated for a zero reading with the calibration thermocouple in the ice bath. The temperature readings at the three thermocouple locations in the sample were observed until the temperature readings at all locations were the same, or until the sample reached a uniform, initial temperature.

After the sample attained a uniform temperature, the temperature limiter-controller was switched on to raise the heater temperature to 300°F. During this heater warmup period, the heater was contained in the auxiliary heater box. The access ports of the sample container and the auxiliary box were plugged with block insulating material. When the heater reached the preset temperature, the plugs in both boxes were removed; the auxiliary box was quickly positioned at the sample container with both access ports lined up and the ends of the boxes flush; and the heater assembly was immediately inserted into the sample container such that the heater surface pressed snugly against the sample end with sufficient pressure to minimize contact resistance but not tight enough to deform the sample and change the distances between the embedded thermocouples.

During this operation, the thermocouple selector switch was set to permit recording of the temperature rise in the sample at the thermocouple location nearest the heated surface, i.e., the surface-temperature function. The temperature at this surface location was recorded continuously except when the thermocouple selector switch was positioned to obtain readings at the other two thermocouple locations. This switching was done every two minutes and readings were taken for twenty seconds at each of these two locations. The temperature history data at the three thermocouple locations were collected for thirty to forty-five minutes. After this duration of time, the heater assembly was removed from the sample container and the data run was concluded.

CHAPTER IV

CALCULATION PROCEDURE

Values of thermal diffusivity were first calculated by a simple procedure to establish at least an estimate of the order of magnitude of the diffusivity of the propellant sample. In an effort to evaluate the accuracy of the results obtained by the simplified calculation procedure, a trial and error solution was employed using a mathematical model similar to the actual experimental situation and a solution which satisfied, as nearly as possible, the real boundary conditions.

The Simplified Calculation Procedure.--By this method the sample was assumed to be the core of an infinite slab. Jakob (1), Schneider (2) and others discuss a method for determining thermal diffusivity of an infinite slab from the temperature response at some point in the slab to a step-function surface temperature elevation from the initial uniform temperature of the sample to a new, higher temperature. The surface temperature remains constant at this new temperature for the duration of the determination. The temperature response at some distance, X, from the surface temperature location can be shown to be an error function of an expression which includes thermal diffusivity, i.e.,

$$\frac{t_x - t_i}{t_s - t_i} = \operatorname{erfc}\left(\frac{X}{2\sqrt{\alpha\tau}}\right) \qquad (1)$$

The response temperature at a known distance in the sample from the surface-temperature thermocouple was measured for various durations of time. With these data, the complementary error function was calculated. Since values of the complementary error function are tabulated in several sources such as Schneider (2) and Carslaw and Jaeger (4), the function term $\frac{x}{2\sqrt{\alpha\tau}}$ and, hence, thermal diffusivity were determined. Figure 7 gives values of the complementary error function for corresponding values of $\frac{x}{2\sqrt{\alpha\tau}}$.

The surface temperature functions of the propellant samples were not step functions due to the extreme difficulty of effecting a sudden elevation of the surface to a new, constant temperature. As a result, an experimental innovation of Equation (1) was used. An average value of surface temperature was substituted for the step-function surface temperature, t_s. This average temperature was obtained at each time point on the response-temperature data curve for which a diffusivity calculation was made. To obtain this average surface temperature at some time point, τ, the corresponding surface temperature curve for the data run was graphically integrated from the start of the run, or $\tau = 0$, up to the calculation point of time τ. This area under the surface-temperature curve for the calculation interval was divided by τ and the resulting temperature value was used in Equation (1). Values of thermal diffusivity for each sample data run were calculated by this simplified procedure at several time points.

<u>The Exact-Solution Calculation Procedure</u>.--The actual experimental heat transfer process occurring during a data run was analyzed and was determined to be essentially unidirectional, unsteady-state conduction

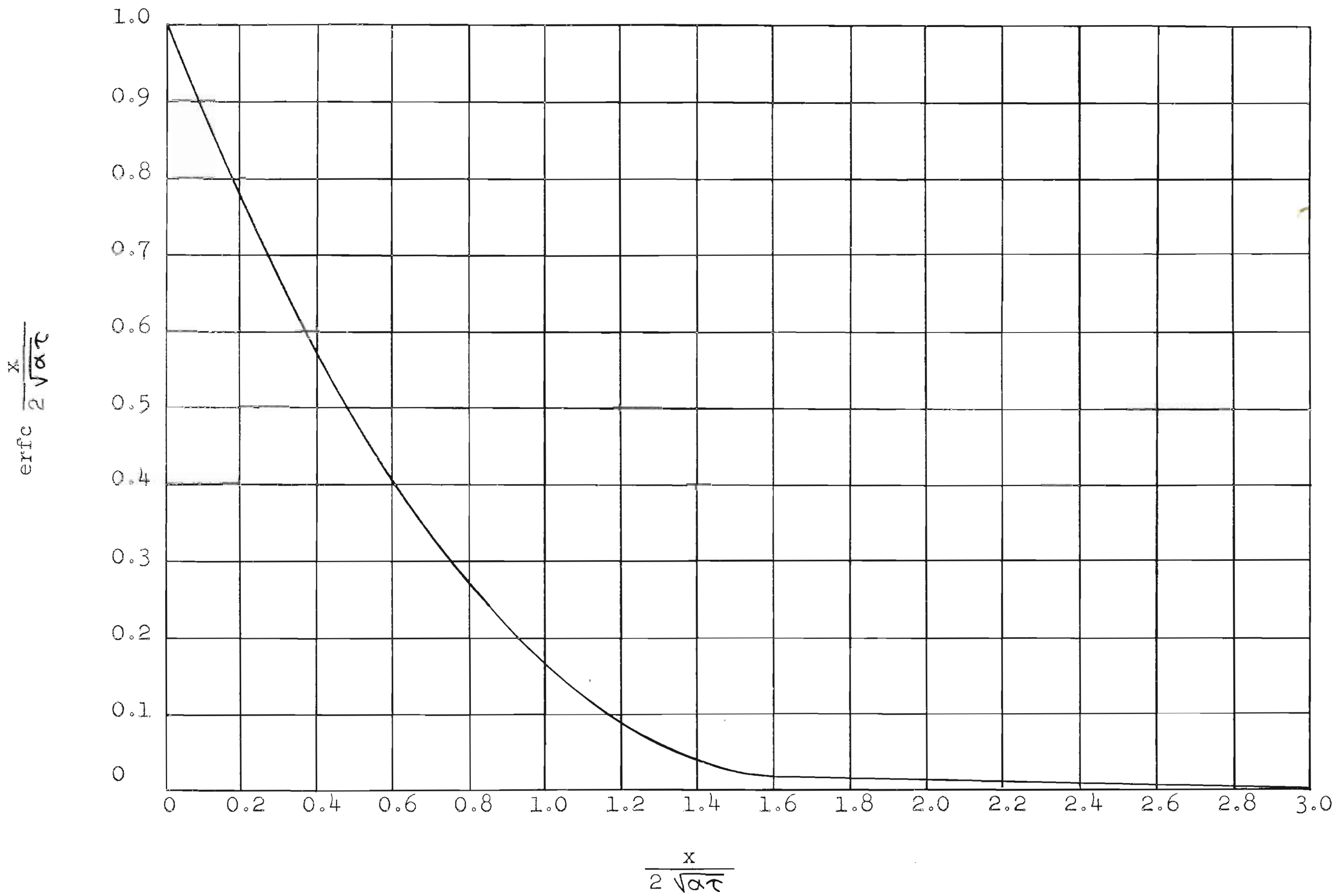


Figure 7. Values of the Complementary Error Function.

heat transfer in a semi-infinite slab. In addition, one end surface remained at the constant, initial uniform temperature of the slab and the other end surface was exposed to a temperature rise which was some function of time. Churchill (5) presents a generalized solution based on Duhamel's theorem for these boundary conditions and for any known transient surface temperature function. The surface temperature functions for the sample runs were all found to be of the form

$$t_s = A - Be^{-C\tau} \quad \text{or} \quad \Theta_s = A - Be^{-C\tau} - t_i \tag{2}$$

Substituting this surface temperature function into Churchill's version of Duhamel's equation resulted in a response temperature equation for the temperature response at any distance x which may be written

$$\theta(x,\tau) = \frac{B(L-x)}{L}\left(1-e^{-C\tau}\right) + \frac{2BC}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\frac{\left(e^{\frac{-\pi^2 n^2\alpha\tau}{L^2}}-e^{-C\tau}\right)}{\left(C-\frac{\pi^2 n^2\alpha}{L^2}\right)}\sin\frac{n\pi(L-x)}{L} \tag{3}$$

A complete development of this equation is given in Appendix B.

By use of appropriate constants in Equation (3) corresponding to a particular data run, a trial and error solution was programmed for an IBM 650 Digital Computer using the Bell General Purpose Programming System. This solution was used to determine the value of thermal diffusivity which, when substituted into Equation (3), resulted in response temperature values that agreed with the experimental response temperature data. The initial assumed value of thermal diffusivity which provided a starting point for the trial and error solution was obtained by averaging the thermal diffusivity values calculated at various time points for the data runs by the simplified procedure. The computer program used in these solutions is presented in detail in Appendix C.

CHAPTER V

DISCUSSION OF RESULTS

The experimental data and calculated results are presented in Tables 1 through 10 and Figures 8 through 13. Values of thermal diffusivity were determined for two propellant samples only. Two data runs were obtained for a sample containing forty per cent aluminum. One run was obtained for a sample containing one per cent aluminum and a second run was attempted. During this attempt, however, the sample became unexplainably ignited (the recorder indicated ignition at 250°F) and caused sufficient minor damage to the equipment to make further investigation inadvisable. Comparison runs using low-conductivity materials of known thermal diffusivity were not made since such samples were not available in the required geometric configuration. Materials of high conductivity were not suitable for comparison purposes.

The results obtained for thermal diffusivity of the samples by the simplified calculation procedure are presented in Tables 4 through 6. In the two data runs for sample type PG, containing twenty per cent aluminum, the thermal diffusivity values became smaller and smaller for calculations at increasing time values. This resulted in a large mean deviation of the calculated values from the average of the calculated values. This presented a problem of determining the time range, either earlier or later in the runs, wherein the calculated diffusivity values were most accurate. This problem appeared to be less acute in the calculations for the data run for sample type ND, containing one per cent aluminum. In this case, the mean deviation is comparatively small due to the small variations between calculated diffusivity results. Nevertheless, the approximate optimum range of applicability of the simplified calculation procedure had to be determined in order to provide a greater degree of general usefulness for the simplified method.

The results of the Duhamel solution provided the basis for analysis of the applicable range problem. As shown in Tables 7 through 9 and Figures 11 through 13, the response-temperature values calculated for various diffusivities by the exact-solution method agreed with the experimental response data either in the earlier or later time ranges of the data run but not over the entire time range. Agreement between the calculated response temperatures for some value of thermal diffusivity and the experimental response data should have resulted if the boundary conditions assumed for the calculation model had been the same as those for the experimental system. As is the case in many heat transfer studies with sample materials of low thermal conductivity and diffusivity, considerable difficulty is experienced in obtaining insulating materials with which to surround the samples to minimize heat losses. Often, the insulating materials which provide the optimum flexibility of handling have conductivity and diffusivity values not much lower than those of the samples. Thus, radial heat-loss sacrifices must be endured to obtain an experimental method providing speed and flexibility for handling large quantities of samples in a short period of time. This, of course, results in inaccuracies of varying magnitude. This appears to be the situation in the experimental system of this investigation.

As a result of the radial heat loss, the assumption that these heat losses would be less pronounced during the earlier period of the data run than at the later time periods is reasonable. Another factor contributing to this delay in radial heat loss effects could have been the method by which the samples were heated. The long rod heater not only heated the sample but also the portion of the insulating material that overlapped the sample. It is possible that the slight resultant rise in temperature of the insulating material at the beginning of a run would have a guard-heater effect during the earlier period of a data run.

In view of these factors, the response-temperature values calculated by the exact-solution method which provided best agreement with experimental data during the initial period of a run were selected as most valid. The respective assumed thermal diffusivity value which resulted in these response temperatures was taken to be the property value for the sample. Further, the initial regions in Figures 11 through 13 in which the calculated response curve for the selected diffusivity values coincide with the experimental data curves were designated as the regions in which the optimum boundary-condition agreement occurred and, hence, the regions of optimum applicability of the simplified calculation procedure. A comparison of the thermal diffusivity values calculated by the exact-solution method with the diffusivity values calculated by the simplified method within the apparent regions of applicability predicted by the exact solution is presented in Table 10. The agreement between results calculated by the two methods is within 10 per cent for all runs. Further, the agreement between the diffusivity values calculated by the exact solution for data runs 1 and 2 for the same sample, type PG, is within 5 per cent. The thermal properties of this type of propellant have not been determined with certainty by other methods and no basis of comparison is available to ascertain the accuracy of the method used in this study. The contributions of instrument response errors, radial heat losses and calculation errors probably account for the deviation between results of two runs for the same sample. The results obtained by the simplified procedure employed in a range of time not exceeding five minutes after the start of the temperature rise at the response-thermocouple location will probably be within 10 per cent of the values predicted by the exact solution method.

Due to certain limitations imposed upon this investigation, extensive data on rocket propellants of the same type and on materials whose diffusivity values are fairly well known were not obtained. As a result, no generalized statement of the accuracy, applicability or over-all value of this method can be made. The method used in this study is rapid but its limitations depend upon the similarity between the experimental boundary conditions maintained and those of the calculation model. Further investigation seems necessary to determine the severity of these limitations.

Table 1. Experimental Data for Run No. 1
Sample Type PG - 20% Aluminum
($X = 0.864$ inches; $t_i = 70°F$)

Time Min.	Surface t_s, mv.	Temperature t_s, °F	Response t_x, mv.	Temperature t_x, °F
0	0.84	70	0.84	70
1	3.82	200	0.84	70
2	4.44	227	0.93	74
3	4.64	236	1.10	82
4	4.80	243	1.29	90
5	4.92	248	1.44	97
6	5.01	252	1.61	104
7	5.03	253	1.74	110
8	5.06	254	1.86	115
9	5.08	255	1.97	120
10	5.10	256	2.07	124
11	5.10	256	2.16	128
12	5.13	257	2.22	131
13	5.15	258	2.32	135
14	5.17	259	2.39	138
15	5.19	260	2.43	140
16	5.19	260	2.50	143
17	5.21	261	2.55	145
18	5.24	262	2.59	147
19	5.24	262	2.64	149
20	5.26	263	2.66	150
21	5.28	264	2.69	151

Table 2. Experimental Data for Run No. 2
Sample Type PG - 20% Aluminum
(X = 0.864 inches; t_i = 76°F)

Time Min.	Surface t_s, mv.	Temperature t_s, °F	Response t_x, mv.	Temperature t_x, °F
0	0.97	76	0.97	76
1	4.07	211	0.97	76
2	4.53	231	1.09	81
3	4.71	239	1.22	87
4	4.85	245	1.38	94
5	4.92	248	1.53	101
6	4.94	249	1.67	107
7	5.01	252	1.81	113
8	5.03	253	1.93	118
9	5.10	256	2.02	122
10	5.13	257	2.11	126
11	5.15	258	2.20	130
12	5.17	259	2.27	133
13	5.19	260	2.34	136
14	5.19	260	2.41	139
15	5.21	261	2.48	142
16	5.21	261	2.52	144
17	5.24	262	2.57	146
18	5.24	262	2.61	148
19	5.26	263	2.66	150
20	5.26	263	2.71	152
21	5.28	264	2.75	154

Table 3. Experimental Data for Run No. 3
Sample Type ND - 1% Aluminum
($X = 0.972$ inches; $t_i = 71°F$)

Time Min.	Surface t_s, mv.	Temperature t_s, °F	Response t_x, mv.	Temperature t_x, °F
0	0.86	71	0.86	71
1	3.15	171	0.86	71
2	4.34	223	0.86	71
3	4.69	238	0.86	71
4	4.85	245	0.86	71
5	4.99	251	0.88	72
6	5.08	255	0.90	73
7	5.15	258	0.95	75
8	5.19	260	1.00	77
9	5.26	263	1.04	79
10	5.28	264	1.09	81
11	5.28	264	1.13	83
12	5.30	265	1.20	86
13	5.30	265	1.27	89
14	5.33	266	1.31	91
15	5.33	266	1.40	95
16	5.35	267	1.45	97
17	5.35	267	1.49	99
18	5.35	267	1.53	101
19	5.37	268	1.58	103
20	5.37	268	1.63	105
21	5.37	268	1.65	106

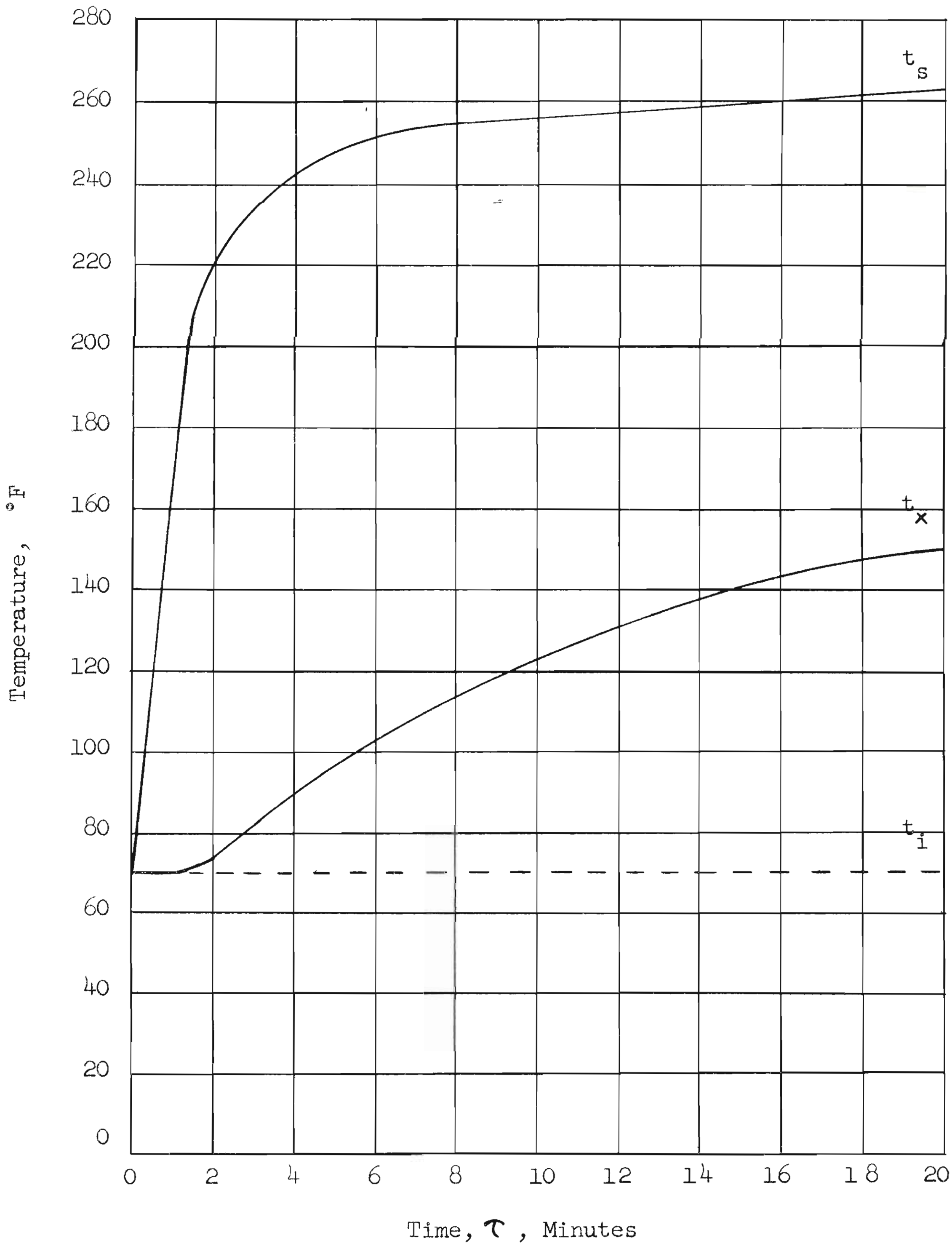


Figure 8. Surface and Response Temperature versus Time Data for Run No. 1.

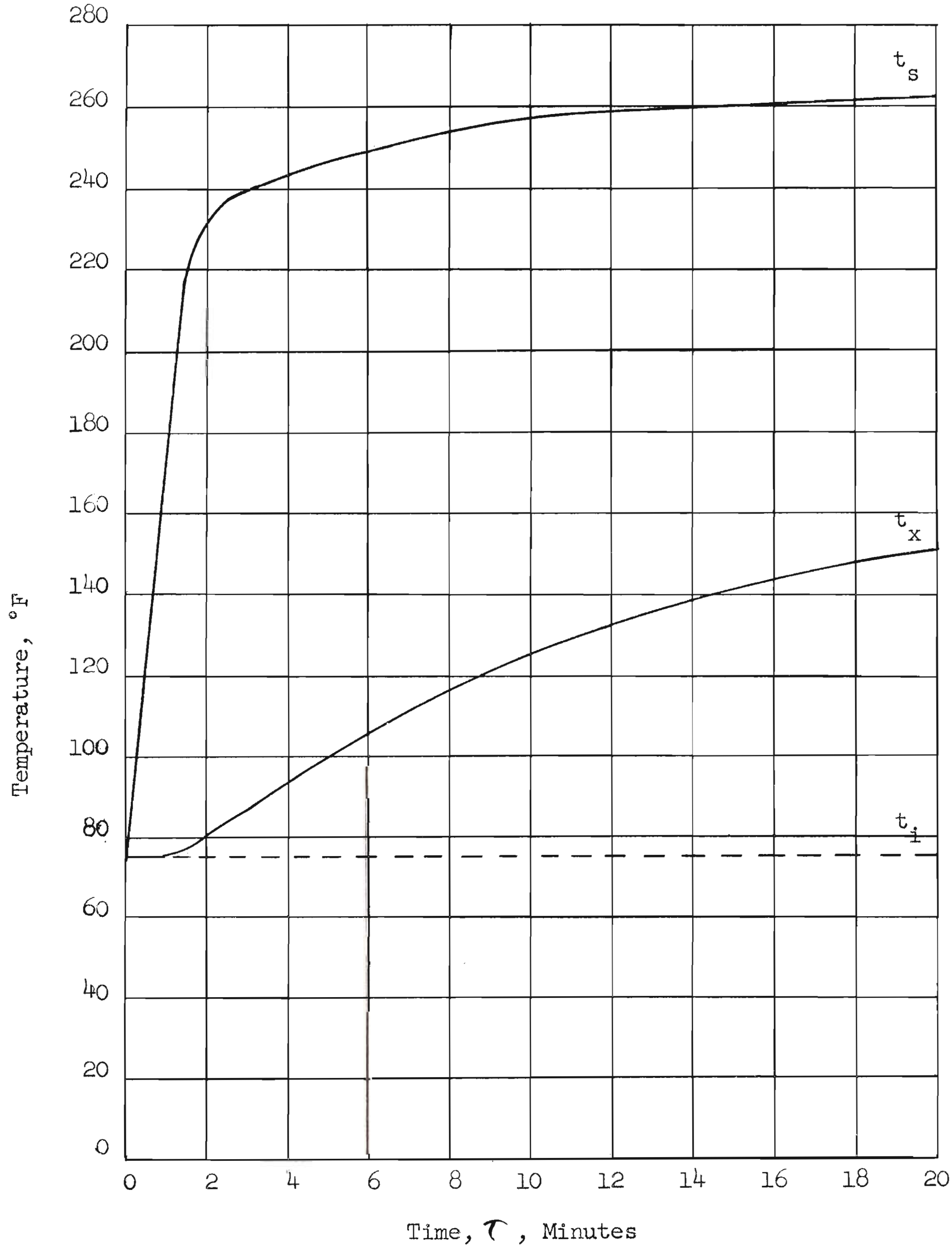


Figure 9. Surface and Response Temperature versus Time Data for Run No. 2.

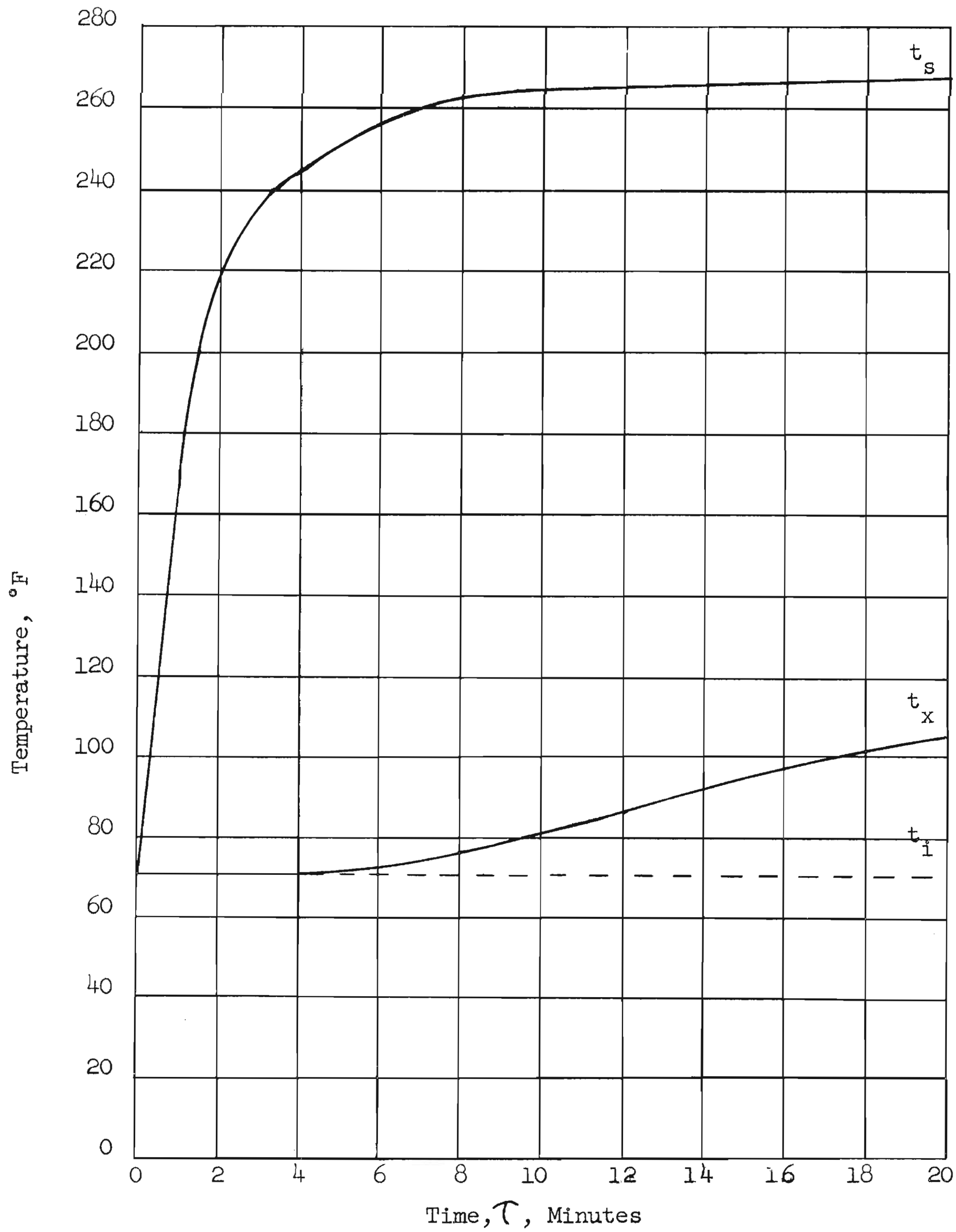


Figure 10. Surface and Response Temperature versus Time Data for Run No. 3.

Table 4. Summary of Diffusivity Calculations
by Simplified Error Function Method
for
Run No. 1, Sample Type PG, 20% Aluminum
($X = 0.864$ inches; $t_i = 70°F$)

τ Time Min.	t_s °F	t_x °F	t_s(avg.) °F	erfc $\frac{x}{2\sqrt{\alpha\tau}}$	α $ft^2/hr.$
4	243	90	207	0.146	18.3×10^{-3}
8	254	115	239	0.281	16.7×10^{-3}
12	257	131	239	0.361	15.5×10^{-3}
16	260	143	244	0.420	14.9×10^{-3}
20	263	150	248	0.450	13.6×10^{-3}

Average $\alpha = 15.8 \times 10^{-3}$ $ft^2/hr.$

Mean Deviation = $\pm$ 8.6%

Table 5. Summary of Diffusivity Calculations by Simplified Error Function Method for Run No. 2, Sample Type PG, 20% Aluminum ($X = 0.864$ inches; $t_i = 76°F$)

τ Time Min.	t_s °F	t_x °F	t_s(avg.) °F	erfc $\frac{x}{2\sqrt{\alpha\tau}}$	α $ft^2/hr.$
4	245	94	212	0.132	17.1×10^{-3}
8	253	118	232	0.269	15.7×10^{-3}
12	259	133	239	0.350	14.6×10^{-3}
16	261	144	244	0.405	13.9×10^{-3}
20	263	151	248	0.436	12.8×10^{-3}

Average $\alpha = 14.8 \times 10^{-3} ft^2/hr.$

Mean Deviation $= \pm 8.5\%$

Table 6. Summary of Diffusivity Calculations by Simplified Error Function Method for Run No. 3, Sample Type ND; 1% Aluminum ($X = 0.972$ inches; $t_i = 71°F$)

τ Time Min.	t_s °F	t_x °F	t_s(avg.) °F	erfc $\frac{x}{2\sqrt{\alpha\tau}}$	α $ft^2/hr.$
6	255	73	216	0.0138	5.72×10^{-3}
8	260	77	227	0.0380	5.52×10^{-3}
12	265	86	239	0.0893	5.68×10^{-3}
16	267	97	245	0.1490	5.90×10^{-3}
20	268	105	249	0.1910	5.68×10^{-3}

Average $\alpha = 5.70 \times 10^{-3} ft^2/hr.$

Mean Deviation = $\pm$ 1.5%

Table 7. Summary of Response Temperature Values for Various Assumed Values of Thermal Diffusivity Calculated by the Exact-Solution Procedure - Run No. 1 (Sample Type PG - 20% Aluminum)

Time τ Minutes	Experimental Data t_x, °F	Calculated Response Temperatures for Assumed α Values		
		$\alpha = 19 \times 10^{-3} ft^2/hr$ t_x, °F	$\alpha = 20 \times 10^{-3} ft^2/hr$ t_x, °F	$\alpha = 21 \times 10^{-3} ft^2/hr$ t_x, °F
0	70	70	70	70
4	90	88	90	91
8	115	121	123	125
12	131	141	144	146
16	143	155	157	160
20	150	164	166	168

Table 8. Summary of Response Temperature Values for Various Assumed Values of Thermal Diffusivity Calculated by the Exact-Solution Procedure - Run No. 2 (Sample Type PG - 20% Aluminum)

Time τ Minutes	Experimental Data t_x, °F	Calculated Response Temperature for Assumed α Values: $\alpha = 18 \times 10^{-3} ft^2/hr$ t_x, °F	$\alpha = 19 \times 10^{-3} ft^2/hr$ t_x, °F	$\alpha = 20 \times 10^{-3} ft^2/hr$ t_x, °F
0	76	76	76	76
4	94	92	94	95
8	118	123	125	127
12	133	143	146	148
16	144	156	158	161
20	152	165	167	170

Table 9. Summary of Response Temperature Values
for Various Assumed Values of Thermal Diffusivity
Calculated by the Exact Solution Procedure - Run No. 3
(Sample Type ND - 1% Aluminum)

Time τ Minutes	Experimental Data t_x, °F	Calculated Response Temperatures for Assumed α Values		
		$\alpha = 5.7 \times 10^{-3} ft^2/hr$ t_x, °F	$\alpha = 6.0 \times 10^{-3} ft^2/hr$ t_x, °F	$\alpha = 6.3 \times 10^{-3} ft^2/hr$ t_x, °F
0	71	71	71	71
4	71	71	71	71
8	77	76	77	78
12	86	86	87	89
16	97	96	99	100
20	105	106	109	111

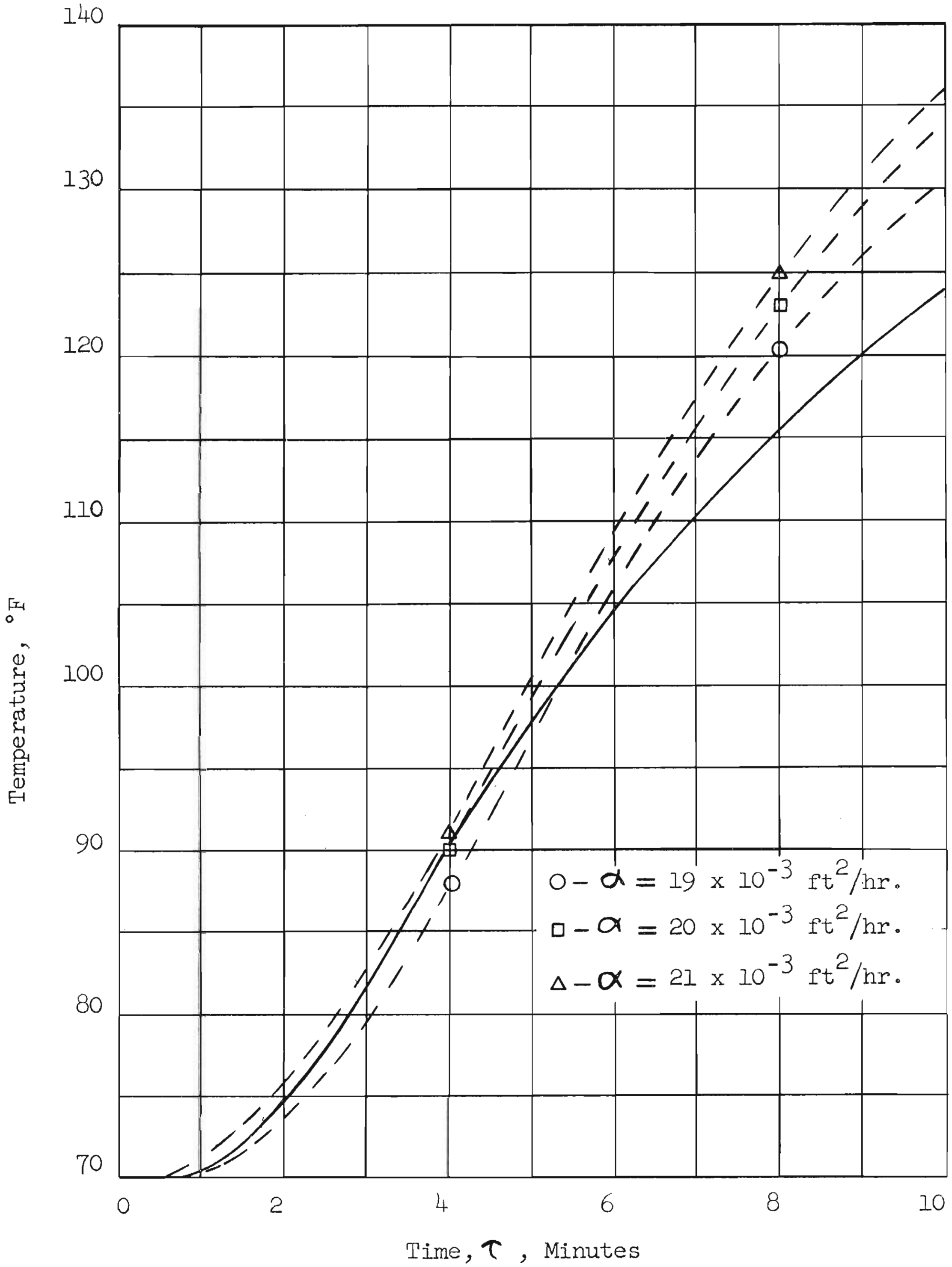


Figure 11. Comparison of Response Temperature Values Calculated by the Exact-Solution Method for Various Values of Diffusivity with Experimental Response Temperature Data - Run No. 1.

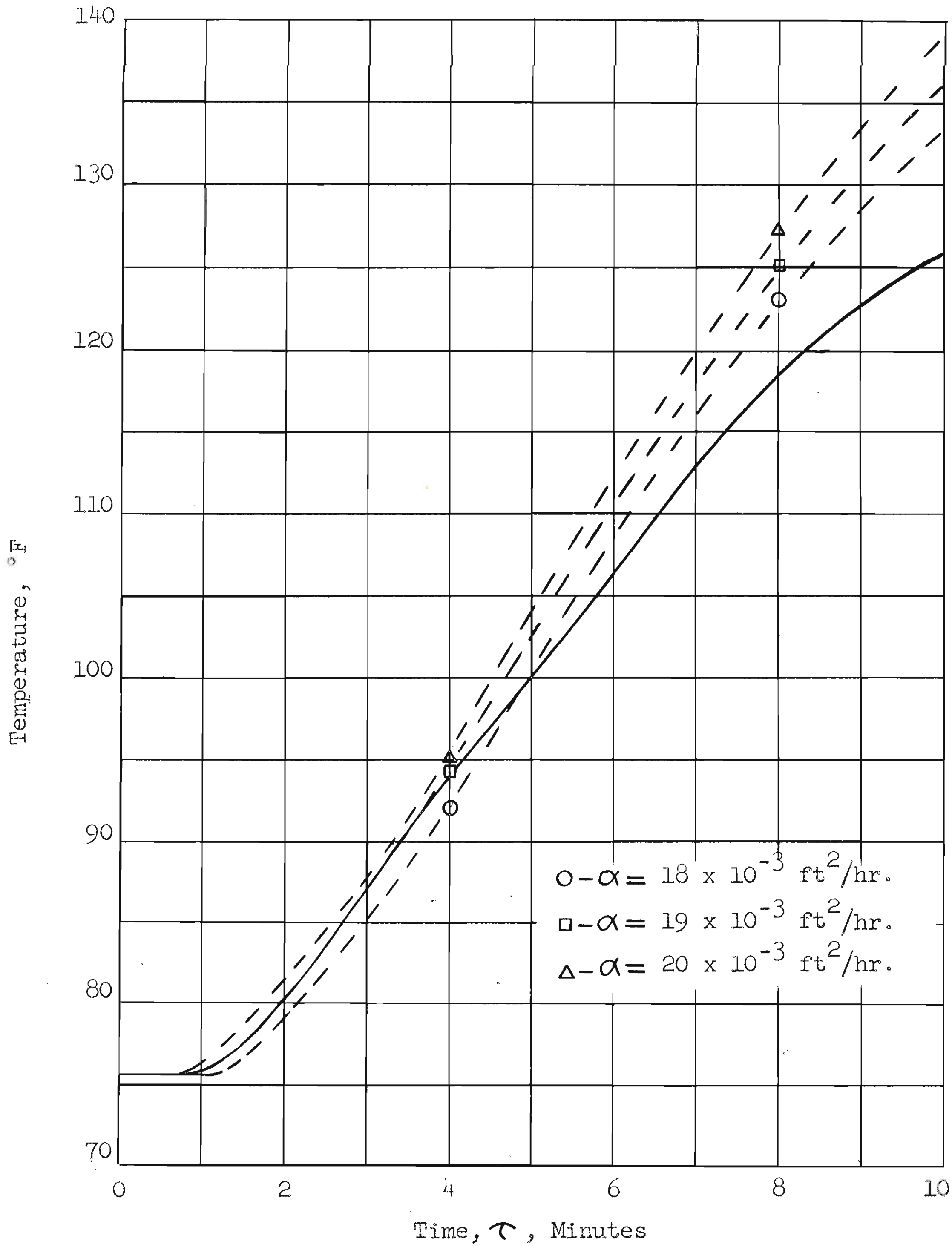


Figure 12. Comparison of Response Temperature Values Calculated by the Exact-Solution Method for Various Values of Diffusivity with Experimental Response Temperature Data - Run No. 2.

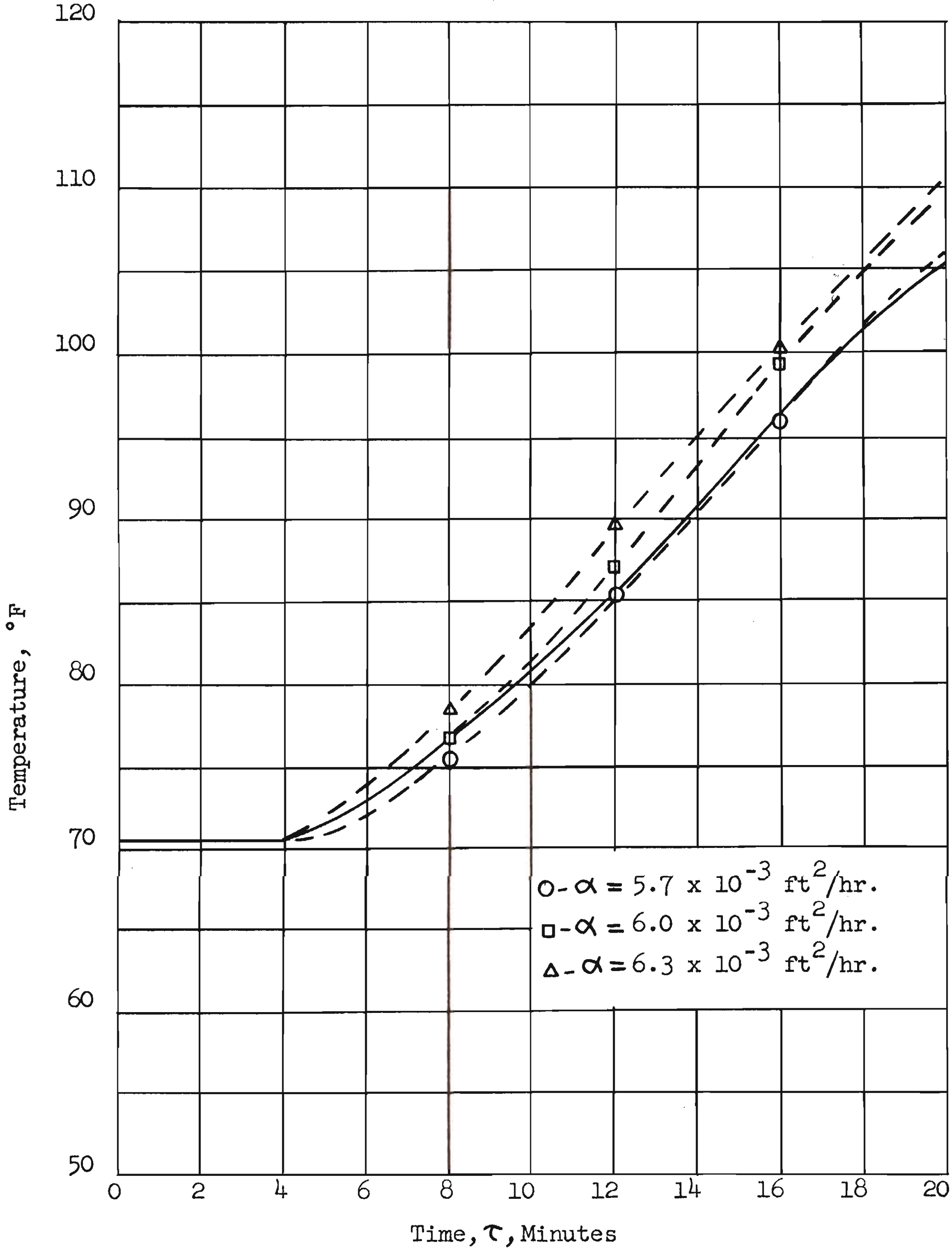


Figure 13. Comparison of Response Temperature Values Calculated by the Exact-Solution Method for Various Values of Diffusivity with Experimental Response Temperature Data - Run No. 3.

Table 10. Comparison of Thermal Diffusivity Results Calculated by the Exact Solution and the Simplified Method in the Predicted Ranges of Applicability

Run No.	Sample Type	Calculated Thermal Diffusivity Results: Exact Solution (ft^2/hr)	Simplified Method (ft^2/hr)	Applicable Range (min)	Agreement Per Cent
1	PG	20.0×10^{-3}	18.3×10^{-3}	1-6	8.5
2	PG	19.0×10^{-3}	17.3×10^{-3}	1-6	9.5
3	ND	6.0×10^{-3}	5.7×10^{-3}	4-9	5.0

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

The conclusions resulting from this investigation may be summarized as follows:

1. The simplified procedure and simplified calculation method described provide a rapid method of determining an approximate thermal diffusivity value for solid rocket propellants such as those used during this investigation.

2. The accuracy of the thermal diffusivity values obtained by the methods used in this study depends upon the similarity between the experimental boundary conditions and those of the calculation model.

3. Under the limitations imposed by the conditions existing during this investigation, the results obtained by the simplified calculation procedure (within a range of time not exceeding five minutes after the start of the temperature rise at the response location) are within 10 per cent of the results obtained by the exact-solution procedure.

4. The diffusivity values calculated by the exact solution provided reproducibility within 5 per cent.

The recommendation is made that additional investigations be conducted using techniques such as guard heaters or guarded hot plates *for heaters or insulating materials of lower conductivity or diffusivity* to reduce radial heat losses and improve the similarity between the experimental and calculation-model boundary conditions. This should greatly improve the accuracy of the methods described for the present investigation.

Further, investigations of the type recommended above should include data runs for materials whose thermal diffusivity values are fairly well known to provide a basis for evaluation of the methods described for the present investigation. If such materials are not available in the desired geometric configurations, comparisons can be made using materials with unreported thermal properties. Of necessity then would be the obtaining of diffusivity values for comparison by measuring separately the individual properties, thermal conductivity, density and specific heat, by methods such as those described by Jakob (1), Giedt (6) or the American Society for Testing Materials (7).

APPENDICES

APPENDIX A

NOMENCLATURE

Capital Letters

A	Constant in the surface temperature equation.
B	Constant in the surface temperature equation.
C	Constant in the surface temperature equation.
F	Temperature function of time.
L	Length of experimental sample, ft.
U	Temperature of a solid at any time and distance in the general temperature response equation.
V	Temperature response in a solid corresponding to a unit step function surface-temperature rise.
X	Distance in sample of response temperature location from heated surface, ft.

Lower Case Letters

k	Number of time increment increases used in computer program (integer).
n	Number of reiterations for series summations in the exact equation (integer).
t	Temperature, °F.

Subscripts

i	Initial conditions at $\tau = 0$.
k	Condition for some value of k.
s	Condition at heated surface of sample.

Subscripts (cont.)

x	Condition at some distance X in sample from heated surface.
0, 1, 2, - - -, n	Condition at some position in the sequence of events, 0 to n, initial to final.

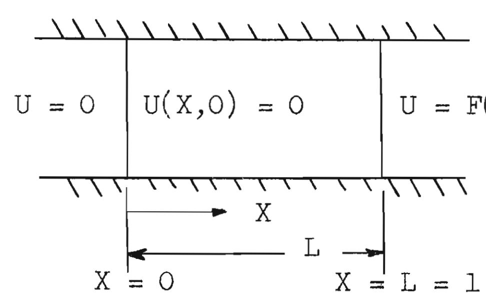
Greek Letters

α	Thermal diffusivity, ft^2/hr.
$\Delta\alpha$	Incremental change in α in computer program.
θ	Difference between temperature in sample, at any distance and time, and the initial, uniform temperature of sample, °F.
λ	Dummy variable in Duhamel equation.
ψ	Defined as e^{-C} in least squares determination of surface temperature function.
τ	Time, min. or hrs. (specified).
$\Delta\tau$	Incremental time change in computer program.

APPENDIX B

DEVELOPMENT OF EQUATION

The exact-solution equation used in the second phase of the calculations of this thesis was developed from the Duhamel formula. Churchill (5) presents a development of a general equation for the temperature response at any distance X and any time τ in a semi-infinite solid having the following boundary conditions:



(a) $U(+0, \tau) = 0$

(b) $U(1, \tau) = F(\tau)$

(c) $U(X, +0) = 0$

Here, U represents the temperature in the solid at any X and τ prescribed.

From a modification of Duhamel's formula, the general equation for temperature at any distance and time is given as

$$U(X,\tau) = F(+0)V(X,\tau) + \int_0^{\tau} F'(\tau - \lambda)V(X,\lambda)d\lambda \qquad (4)$$

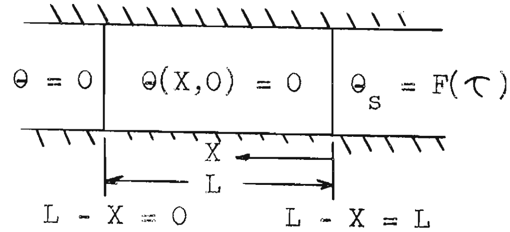
$V(X,\tau)$ is the temperature response corresponding to a unit step function surface-temperature rise, i.e., $F(\tau) = 1$ for $+0 < \tau < \infty$. λ represents a dummy variable. For unit thermal diffusivity and unit length

$$V(X,\tau) = X + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{-n^2\pi^2\tau}\sin n\pi X . \tag{5}$$

Substituting equation (5) into equation (4),

$$U(X,\tau) = X F(\tau) + \frac{2F(+0)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{-n^2\pi^2}\sin n\pi X + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin n\pi X\int_0^{\tau}F'(\tau-\lambda)e^{-n^2\pi^2\lambda}d\lambda \tag{6}$$

Churchill's development, as presented above, may be applied to the analysis of this investigation by letting $\Theta = U$, where $\Theta = t_x - t_i$, the difference between the temperature at some point X and the initial, uniform temperature of the sample. Since X in the experimental procedure was measured from the heated end of the sample, the X of Churchill's notation will be replaced by $(L - X)$. The boundary conditions now become



$$\text{(a)}' \quad \theta(+0, \tau) = 0$$

$$\text{(b)}' \quad \theta(L, \tau) = F(\tau)$$

$$\text{(c)}' \quad \theta(X, +0) = 0$$

Since the boundary conditions are essentially identical to those of Churchill's development, the modified Duhamel formula, equation (4), may be written

$$\theta(x,\tau) = F(+0)\,V(x,\tau) + \int_0^{\tau} F'(\tau-\lambda)\,V(x,\lambda)\,d\lambda \tag{7}$$

For length and diffusivity not equal to unity, equation (6) becomes

$$\theta(x,\tau) = \frac{(L-x)}{L}F(\tau) + \frac{2F(+0)}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n} e^{\frac{-n^2\pi^2\alpha\tau}{L^2}} \sin\frac{n\pi(L-x)}{L} \tag{8}$$

$$+ \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi(L-x)}{L}\int_0^{\tau} F'(\tau-\lambda)\,e^{\frac{-n^2\pi^2\alpha\lambda}{L^2}}\,d\lambda$$

From the experimental surface temperature data, the surface temperature function, $F(\tau)$, was found to be of the form

$$t_s = A - Be^{-C\tau}$$

or

$$F(\tau) = \theta_s = B(1 - e^{-C\tau}) \tag{9}$$

Then, $F(+0) = 0$ (10)

and $F'(\tau - \lambda) = BCe^{-C(\tau-\lambda)}$ (11)

Substituting equations (9), (10) and (11) into equation (8) results in

$$\theta(x,\tau) = \frac{B(L-x)}{L}\left(1 - e^{-c\tau}\right) + \frac{2BC}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi(L-x)}{L}\, e^{-c\tau}\int_0^{\tau} e^{\left(c - \frac{n^2\pi^2\alpha}{L^2}\right)\lambda}\, d\lambda \tag{12}$$

Integrating the last term gives

$$\theta(x,\tau) = \frac{B(L-x)}{L}\left(1 - e^{-c\tau}\right) + \frac{2BC}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\frac{\left[e^{-\frac{n^2\pi^2\alpha\tau}{L^2}} - e^{-c\tau}\right]}{\left[c - \frac{n^2\pi^2\alpha}{L^2}\right]}\sin\frac{n\pi(L-x)}{L} \tag{13}$$

This equation is an exact solution for Θ and, hence, t_x at any location in the sample and at any time in the interval $+0 < \tau < \infty$.

APPENDIX C

METHOD OF CALCULATION

A. Evaluation of Constants in Surface Temperature Equation

By a method of least squares, the experimental data of the form

τ (hours)	t_s (°F)
τ_o	t_o
τ_1	t_1
τ_2	t_2
.	.
.	.
.	.
τ_n	t_n

is fitted to an equation of the form

$$t_s = A - Be^{-C\tau} \tag{14}$$

1. To determine the constant C:

$$\text{Let } e^{-C} = \psi$$

Neglecting the τ_o, t_o point:

$$
\begin{aligned}
(t_2 - t_1)\psi &= t_3 - t_2 \\
(t_3 - t_2)\psi &= t_4 - t_3 \\
&\vdots \\
(t_{n-1} - t_{n-2})\psi &= t_n - t_{n-1} \\
\hline
\sum_3^n (t_{n-1} - t_{n-2})\psi &= \sum_3^n (t_n - t_{n-1})
\end{aligned}
$$

Then,

$$\psi = \frac{\sum_3^n (t_n - t_{n-1})}{\sum_3^n (t_{n-1} - t_{n-2})} \tag{15}$$

Since $\quad \psi = e^{-C}$

Then $$C = -\ln \psi \tag{16}$$

2. The constant A and B are determined simply by a least squares method for a two constant polynomial equation.

B. Evaluation of constants in the Θ_s equation

Since an equation of the form

$$t_s = A - Be^{-C\tau}$$

was found for the surface temperature equation from the experimental data of a run, then

$$\Theta_s = t_s - t_i = A - Be^{-C\tau} - t_i \tag{17}$$

where $\quad t_i = t_0$, or t_s at $\tau = 0$

But in equation (14), for $\tau = 0$ and $t_s = t_o = t_i$,

$$t_i = A - B.$$

Hence

$$A = B + t_i. \qquad (18)$$

Substituting equation (18) into equation (17),

$$\Theta_s = B + t_i - Be^{-C\tau} - t_i = B - Be^{-C\tau}.$$

Therefore,

$$\Theta_s = B(1 - e^{-C\tau}) \qquad (19)$$

C. Solution of Exact Equation

With values of B and C determined for the Θ_s function for a particular data run, the reiterative solution of the exact equation was programmed for the IBM 650 digital computer of the Rich Electronic Computer Center using the Bell General Purpose System of programming. The program provided for a trial-and-error solution of the exact equation for thermal diffusivity by calculating response-temperature values at various periods of time (at an X value corresponding to the location of the response thermocouple in the experimental sample) for assumed values of thermal diffusivity. The answer cards were monitored continuously and the program was stopped when a particular diffusivity gave response temperature-time values most nearly like the experimental response temperature-time data. The average thermal diffusivity value for the data run predicted by the simplified calculation procedure provided the initial assumed diffusivity value. From this value the computer calculated the first response values, then increased the diffusivity value by a prescribed increment, computed another set of response values, and repeated this cyclic operation until the program was stopped manually by the operator. Samples of the data storage locations and the program command sequence are given in Tables 11 and 12, respectively.

Table 11. Data Storage Locations in Computer for Solution Program

Item Stored	Location	Description of Item
B	501	Run constant in Θ_s equation
C	502	Run Constant in Θ_s equation
X	503	Response location in sample
π	504	Constant (3.1416)
-1	505	Multiplier
L	507	Length of sample
α	508	Thermal diffusivity (initial value)
$\Delta\tau$	512	Incremental time change
$\Delta\alpha$	513	Incremental α change

Table 12. Program Command Sequence for a Typical Run

Command No.	Operation Commanded to Computer
1	Establish Program Point 3 for α re-entry.
2	Compute τ_k and store until increased.
3	Establish Program Point 2 for τ re-entry.
4	Compute $(-1)^n$
5	Compute $C\tau$
6	Compute $e^{-C\tau}$
7	Compute $(1 - e^{-C\tau})$
8	Compute $B(1 - e^{-C\tau})$
9	Compute $L - X$
10	Compute $\frac{L - X}{L}$
11	Compute $B \frac{L - X}{L} (1 - e^{-C\tau})$
12	Establish Program Point 1 for n re-entry
13	Compute $\frac{\pi (L - X)}{L}$
14	Compute $\frac{n\pi(L - X)}{L}$
15	Compute $\sin \frac{n\pi(L - X)}{L}$
16	Compute n^2
17	Compute π^2
18	Compute $n^2\pi^2$
19	Compute L^2

Table 12. Program Command Sequence for a Typical Run (Continued)

Command No.	Operation Commanded to Computer
20	Compute $\frac{n^2 \pi^2}{L^2}$
21 & 22	Compute $\frac{n^2 \pi^2 \alpha \tau}{L^2}$
23	Compute $e^{\frac{-n^2 \pi^2 \alpha \tau}{L^2}}$
24	Compute $e^{\frac{-n^2 \pi^2 \alpha \tau}{L^2}} - e^{-C\tau}$
25	Compute $C - \frac{\pi^2 \alpha n^2}{L^2}$
26	Compute $\left(e^{\frac{-n^2 \pi^2 \alpha \tau}{L^2}} - e^{-C\tau}\right) \sin \frac{n\pi(L - X)}{L}$
27	Compute $\dfrac{\left(e^{\frac{-n^2 \pi^2 \alpha \tau}{L^2}} - e^{-C\tau}\right) \sin \frac{n\pi(L - X)}{L}}{\left(C - \frac{n^2 \pi^2 \alpha}{L^2}\right)}$
28	Compute $\dfrac{(-1)^n\left(e^{\frac{-n^2 \pi^2 \alpha \tau}{L^2}} - e^{-C\tau}\right) \sin \frac{n\pi(L - X)}{L}}{n\left(C - \frac{n^2 \alpha \pi^2}{L^2}\right)}$
29	Compute $\dfrac{(-1)^n\left(e^{\frac{-n^2 \alpha \pi^2 \tau}{L^2}} - e^{-C\tau}\right) \sin \frac{n\pi(L - X)}{L}}{n\left(C - \frac{n^2 \alpha \pi^2}{L^2}\right)}$

Table 12. Program Command Sequence for a Typical Run (Continued)

Command No.	
30	Compute BC
31	Compute 2BC
32	Compute $\frac{2BC}{\pi}$
33	Compute $$\frac{2BC(-1)^n\left(e^{\frac{-n^2\alpha\pi^2\tau}{L^2}} - e^{-C\tau}\right)\sin\frac{n\pi(L - X)}{L}}{\pi n\left(C \quad \frac{n^2\alpha\pi^2}{L^2}\right)}$$
34	Compute $\sum(\Theta_x)_n$
35	Increase n by 1
36	Repeat from Program Point No. 1 n times.
37	Punch out τ_k, α and Θ_x
38	Increase τ_k by $\Delta\tau$
39	Repeat from Program Point No. 2 k times
40	Increase α by $\Delta\alpha$
50	Repeat from Program Point No. 3 (Manually stopped by operator)

BIBLIOGRAPHY

1. Jakob, Max, Heat Transfer, Vol. 1, New York: John Wiley and Sons, 1949.

2. Schneider, P. J., Conduction Heat Transfer, Cambridge, Mass: Addison-Wesley Publishing Company, Inc., 1955.

3. Chung, Paul K., and Melbourne L. Jackson, "Thermal Diffusivity of Low Conductivity Materials," Industrial and Engineering Chemistry 46, 2563-66, (1954).

4. Carslaw, H. S., and J. C. Jaeger, Conduction of Heat in Solids, London: Oxford University Press, 1947.

5. Churchill, Ruel V., Modern Operational Mathematics in Engineering, New York: McGraw-Hill Book Company, Inc., 1944.

6. Giedt, Warren H., Principles of Engineering Heat Transfer, Princeton, New Jersey: D. Van Nostrand Company, Inc., 1957.

7. Method C 177-45, Philadelphia: American Society for Testing Materials.